VIITJEE ACADEMY

PHYSICS(JR) IMPORTANT FORMULAE

1.**PARALLELOGRAM LAW OF VECTORS:**

a)Magnitude of resultant is $|\vec{R}|=\sqrt{P^2+Q^2+2PQ\cos\theta}$

b)Direction of resultant : $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)$

2.***MOTION OF A BOAT CROSSING THE RIVER IN SHORTEST TIME :***

a) the boat is to be rowed across the river i.e., along normal to the banks of the river.

b)Time taken to cross the river, $t=\frac{d}{V_B}$

c)The distance (drift)(BC) travelled downstream

$=d \times \frac{V_R}{v_B}$

3.MOTION OF A BOAT CROSSING THE RIVER IN SHORTEST DISTANCE :

a) The boat is to be rowed upstream making some angle 'θ with normal to the bank of the river which is given by $\theta = \sin^{-1}\left(\frac{V_R}{V_B}\right)$

b) The time taken to cross the river is

$t=\frac{d}{\sqrt{V_B^2-V_R^2}}$

4.**SCALAR PRODUCT or DOT PRODUCT**

a)The dot product of two vectors is a scalar.

The scalar product of two vectors $\bar{a}$ and $\bar{b}$ is

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ where θ is angle beween $\vec{a}$ *and* $\vec{b}$

5.CROSS PRODUCT or VECTOR PRODUCT

a)If $\bar{a}$ and $\bar{b}$ are two vectors and the angle between them is θ then the cross product of $\bar{a}$ and $\bar{b}$ is given by $\vec{a}\times\vec{b} = \left(|\vec{a}||\vec{b}|\ \sin\theta\right)\hat{n}$

where $\hat{n}$ is a unit vector perpendicular to the plane containing $\bar{a}$ and $\bar{b}$.

6.If a body covers first half of its journey with uniform velocity v_1 and the second half of the journey with uniform velocity v_2, then the average velocity

$v = \frac{2v_1v_2}{v_1+v_2}$

a) Ratio of displacements in the 1sts, 2nds, 3rd s... nth s = 1:3:5:.....:(2n-1)

b) Ratio of displacements in the first 1s, first 2s, first 3s .... etc.... is 1:4:9:.... etc.

7.● If a bullet looses $(1/n)^{th}$ of its velocity while passing through a plank, then the no. of such planks required to just stop the bullet is $\frac{n^2}{2n-1}$

8. A body is describing uniform circular motion with a speed 'v'. When it describes an angle 'θ' at the center then the change in velocity is $\Delta v = 2v\sin(\theta/2)$

9. Starting from rest a body travels with an acceleration 'α' for some time and then with deceleration 'β' and finally comes to rest. If the total time of journey is 't', then the maximum velocity and displacement and average velocity are respectivelly

i) $v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$, ii) $s = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$

iii) average velocity = $\left(\frac{v_{max}}{2}\right)$

10.GRAPHS

Characteristics of s-t and v-t graphs

- Slope of displacement-time graph gives velocity
- Slope of velocity-time graph gives acceleration
- Area under velocity-time graph gives displacement
- Area under acceleration-time graph gives change in velocity

11. A particle starts from rest and moves along a straight line with uniform accelaration. If s is the distance travelled by it n seconds and s_n is the distance travelled in the n^{th} second, then

$\frac{s_n}{s} = \frac{(2n-1)}{n^2}$

12.● A stone is dropped into a well of depth 'h', then the sound of splash is heard after a time of 't'

$t = \sqrt{\frac{2h}{g}} + \frac{h}{v_{sound}}$

13. A body projected vertically up crosses a point *P* at a height '*h'* above the ground at time 't_1'

seconds and at time t_2 seconds to same point while coming down. Then total time of its flight $T = t_1 + t_2$

a) Height of *P* is $h = \frac{1}{2} g t_1 t_2$ b) Maximum height reached above the ground

$$H = \frac{1}{8} g (t_1 + t_2)$$

c) Magnitude of velocity while crossing *P* is

$$\frac{g(t_2 - t_1)}{2}$$

14.**Oblique Projectile :**

- Horizontal component of velocity $u_x = u\cos\theta$, remains constant throughout the journey.
- Vertical component of velocity $u_y = u\sin\theta$, varies at the rate of 'g'.
- After a time 't'
  (a) Horizontal component of velocity
  $v_x = u\cos\theta$
  (b) Vertical component of velocity $v_y = u_y - gt = u\sin\theta - gt$

(c) Resultant velocity v = $\sqrt{v_x^2 + v_y^2}$

(d) Direction of velocity is given by $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

(e) Horizontal displacement during a time $t x = u_x t = (u\cos\theta)t$

(f) Vertical displacement during a time t

$y = u_y t - \frac{1}{2} gt^2 = (u\sin\theta)t - \frac{1}{2} gt^2$

(g) Net displacement of the body = $\sqrt{x^2 + y^2}$

(h) Equation of a projectile

$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta} x^2 = Ax - Bx^2$ (i) From the above equation

- $\theta = \tan^{-1}(A)$
- Range of the projectile $R = \frac{A}{B}$
- Maximum height $H = \frac{A^2}{4B}$

a) At the maximum height, the vertical component of velocity becomes zero.

(b) The velocity of the projectile is minimum at the highest point and equal to $u\cos\theta$.

(c) Acceleration is equal to acceleration due to gravity 'g', and it always acts vertically

- Horizantal range

$$R = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin 2\theta}{g}$$

(a) Range is maximum when $\theta = 45^0$

(b) Maximum range, $R_{Max} = \frac{u^2}{g}$

(c)When 'R' is maximum, $H_{Max} = \frac{R_{Max}}{4} = \frac{u^2}{4g}$

(d) $R = \frac{gT^2}{2\tan\theta}$

If $\theta = 45^0$ then $R = \frac{gT^2}{2} \Rightarrow T = \sqrt{\frac{2R}{g}}$

(e) When two bodies are projected with same initial velocity but at two different angles of projection

(i) θ and $(90 - \theta)$ (or) (ii) $(45 - \theta)$ and $(45 + \theta)$ (or) iii) θ with horizontal and θ with vertical

Then If T_1 and T_2 are the times of flight then

a) $\frac{T_1}{T_2} = \tan\theta$ b) $T_1 T_2 = \frac{2R}{g}$

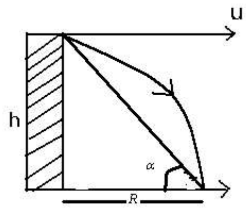
- If H_1 and H_2 are maximum heights then

a) $\frac{H_1}{H_2} = \tan^2\theta$ b) $H_1 + H_2 = \frac{u^2}{2g}$ c) $R = 4\sqrt{H_1 H_2}$

- $4H = R\tan\theta$
- When $R = H$ then $\theta = \tan^{-1}(4) = 76^0$

15.HORIZONTAL PROJECTILE :

- When a body is projected horizontally with a velocity from a point above the ground level, it is called a *Horizantal Projectile.* When a stone is projected horizantally with a velocity 'u' from the top of a tower of height 'h' it describes a parabolic path as shown in *figure.*



a) Time of descent $t = \sqrt{\frac{2h}{g}}$ (independent of *u*)

b) The horinzontal displacement (or) range $R = u\sqrt{\frac{2h}{g}}$

c) The velocity with which it hits the ground

$$V = \sqrt{u^2 + 2gh} = \sqrt{u^2 + g^2t^2}$$

16.• If gravel is dropped on a conveyor belt at the rate of $\frac{dm}{dt}$, extra force required to keep the belt moving with velocity 'u' is $F = u\left(\frac{dm}{dt}\right)$

17.• A ball of mass '*m*' strikes a wall with a velocity 'v' making an angle 'θ' with the **normal to its surface** and bounces with same speed 'v' at same angle θ with the normal, then the change in momentum is 2mv cos θ and is directed away from the surface along the normal.

18. **Atwood's Machine :-**

Masses M_1 and M_2 are tied to a string , which passes over a frictionless light fully . The string is light and inextensible.a) If $M_1 > M_2$, M_1 moves down.and M_2 moves up with common acceleration 'a' then $a = \frac{(M_1 - M_2)g}{M_1 + M_2}$......(3) b.T $= \left(\frac{2M_1M_2}{M_1 + M_2}\right)g$ c. Thrust on the pulley $= 2T = \left(\frac{4M_1M_2}{M_1 + M_2}\right)g$

19. Law of conservation of momentum :

- When a shot is fired from a gun, while the shot moves forwards, the gun moves backwards. This motion of gun is called **recoil of the gun**. When a gun of mass 'M' fires a bullet of mass 'm' with a muzzle velocity 'v', the gun recoils with a velocity 'V' given by V = mv/M.
- When a shot is fired from a gun, the kinetic energies of the shot and gun are in the inverse ratio of their masses. $\frac{K.E. of\ the\ shot}{K.E. of\ the\ gun} = \frac{M}{m}$, where *M* is mass of the gun and *m* is mass of the shot.
- When a bullet of mass 'm' moving with a velocity 'v' gets embedded into a block of mass M at rest and free to move on a smooth horizontal surface, then their common velocity V = mv/ (M + m).
- A boy of mass 'm' walks a distance 's' on a boat of mass 'M' that is floating on water and initially at rest. If the boat is free to move, it moves back a distance = ms / (M + m).
- A shell of mass 'M' explodes into two fragments and one of mass 'm' moves out with a velocity 'v' the other piece of mass (M - m) moves in opposite direction with a velocity of mv/ (M - m).
- When a machine gun fires 'n' bullets each of mass 'm' with a velocity 'v' in a time interval 't' the force needed to hold the gun steady is F = mnv/t.

20. ROCKET PROPULSION:

- Accelaration of rocket $F = u\frac{dM}{dt}$, $a = \frac{u}{M}\frac{dM}{dt}$
- u_0 is initial velocity of rocket and u is the velocity of ejected gas then velocity of rocket at any time $V = u_0 + u\log_e\left[\frac{M_0}{M}\right]$
- If the effect of gravity is considered; thenAcceleration $a = \frac{u}{M}\left(\frac{dM}{dt}\right) - g$

  Force/thrust $F = Ma = u\left[\frac{dM}{dt}\right] - Mg$

21.**WORK :**

- Work done in compressing or expanding a spring: W = ½ K x^2 where 'K' is the force constant of the spring and '*x*' is the extension or compression. Work done in changing the elongation of a spring from x_1 to x_2 is W = ½ K($x_2^2 - x_1^2$)
- A uniform chain of mass 'm' of length '*l*' is hanging $\frac{1}{n}$th from edge of table. Workdone to pull the hanged part on to the table is $\frac{mgl}{2n^2}$.
- The work done in lifting a body of mass 'm' having density d_1 inside a liquid of density d_2 through a height 'h' is

$$W = mgh\left(1-\frac{d_2}{d_1}\right)$$

- A bucket full of water of total mass 'M' is pulled up using a uniform rope of mass 'm' and length 'l', work done is $W = Mgl + mg\frac{l}{2}$

22. **POWER** :

- Instantaneous power: If ΔW is the work done in a time Δt, the instantaneous power,

  It is also calculated by P = F v cos θ or $P = \overline{F}\cdot\overline{v}$

23. Kinetic energy in terms of linear momentum 'p' is given by $K = \frac{1}{2}p\,v$ and $K = \frac{p^2}{2m}$

24. Elastic Collision in one Dimension

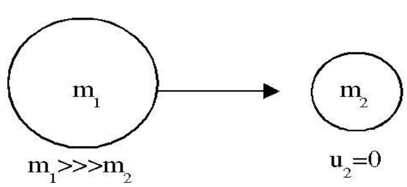
- In a perfect elastic collision coefficient of restitution is "1" ;
- Two bodies of masses m_1 and m_2 moving in the same direction with velocities u_1 & u_2 collide elastically ; The final velocities of the two bodies after collusion

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

Case 1 : If $m_1 = m_2$; $v_1 = u_2$; $v_2 = u_1$;

Case 2 : If $m_1 = m_2$; $u_2 = 0$; $v_1 = 0$; $v_2 = u_1$

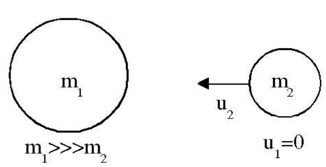
Case 3 :



$v_1 = u_1$; $v_2 = 2u_1$;

i.e., A large body moving with same velocity collides with a lighter body of rest ; Then there is almost no change in the velocity of heavy body ; But lighter body moves with double velocity of large body.

Case 4 :



$v_1 = 0$; $v_2 = -u_1$;

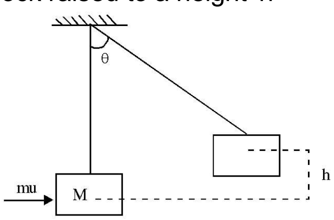
i.e., A lighter body moving with same velocity collided a large body at rest ; Then the large body remaining at rest, but lighter body retrace its path with same velocity.

25. ***PERFECTLY INELASTIC COLLISION***

In a perfectly inelastic collision, the two bodies stick together after impact and move with common velocity, so the coefficient of restitutions e = 0;

26. BALLISTIC PENDULUM

A Block of mass 'M' is suspended by means of a rope 'l' ; A bullet of mass 'm' moving with a velocity 'u' collides with the block and get embedded on it; If the block raised to a height 'h'



mu = (M + m) v

[∵ law of conservation of momentum]

v = $\sqrt{2gh}$ [conservation of energy]

mu = (M+m)$\sqrt{2gh}$ u = $\left(\frac{M+m}{m}\right)\sqrt{2gh}$

⇒ If a body of mass m_1 moving with velocity u_1 collides inelastically with a stationary mass m_2, then the loss in kinetic energy is given by

$$KE = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2\left(1 - e^2\right)$$

In case of perfectly inelastic collision $e = 0$

then $KE = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2$

⇒ If a body of mass m_1 moving with velocity u_1 collides elastically with a stationary mass m_2, then fraction of kinetic energy transfered is given by $\frac{\Delta K.E.}{K.E.} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$

⇒ Fraction of K.E. retained with the first body $1 - \frac{4m_1 m_2}{(m_1 + m_2)^2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

27. **COEFFICIENT OF RESTITUTION**

The ratio of relative velocity of separation after collision to the relative velocities of approach before collision is called coefficient of restitution. $e = \frac{v_2 - v_1}{u_1 - u_2}$

'e' depends on the nature of the materials of the colliding bodies.

→ No units and dimensions → Its value lies between 0 & 1

28. CENTRE OF MASS

→ In a system of two prticles of mass m_1 and m_2, when m_1 is pushed towards m_2 through a distance d then shift in m_2 towards m_1 without alterning C.M position is $\frac{m_1}{m_2}d$.

→I A boy of mass m is at one end of a flat boat of mass M and length l which floats on water. If the body moves to the other end, the boat moves through a distance d in the opposite direction such that $d = \frac{ml}{(M+m)}$.

→ To a circular disc of radius R_1 another of radius R_2 and of the same material is added then shift in the CM is x = $\frac{R_2^2(R_1 + R_2)}{R_1^2 + R_2^2}$

→ From a disc of radius 'R' a disc of radius 'r' is removed from one end. Then shift in its centre of mass isx = $\frac{r^2(R - r)}{R^2 - r^2}$

→ For a semi circular plate of radius 'r' $X_c = \frac{4r}{3\pi}$

→ The centre of mass of a wire bent into the form of a half circle of radius R is $X_c = \frac{2R}{\pi}$

29. CIRCULAR MOTION

- If an electron revolves around the nucleus of hydrogen atom in different circular orbits of radii r_1, r_2 with time periods T_1, T_2 and frequencies υ_1, υ_2 $\boxed{\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \text{ and } \left(\frac{\upsilon_1}{\upsilon_2}\right)^2 = \left(\frac{r_2}{r_1}\right)^3}$

- If n_1, n_2 are principal quantum numbers of orbits of radii r_1, r_2 then $\boxed{\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2}$

- The directions of $\vec{V}, \vec{\omega}$ *and* $\vec{r}$ are mutually perpendicular $\boxed{\therefore V = \omega r}$

- When a body is rotating about its own axis at an angular velocity ω then all particles of the body have the same angular velocity. If V_1, V_2 are linear velocities of particles at perpendicular distances r_1, r_2 from the axis of rotation then $V_1 = r_1\omega$; $V_2 = r_2\omega$; $\boxed{\frac{V_1}{V_2} = \frac{r_1}{r_2}}$

- The particles on the axis of rotation have "Zero" linear velocity.

- A circular disc is rotating in horizontal plane at an angular velocity 'ω' about vertical axis passing through the centre of the disc, and a small coin is placed on the rotating disc at a distance 'r' from the axis of rotation.

  If μ_S is the coefficient of static friction between the coin and the disc, then the condition for the coin just to skid is the coin does not skid

  if $\boxed{\min \mu_s = \frac{r\omega^2}{g}}$ $\boxed{r_{max} = \frac{\mu_s g}{\omega^2}}$

  $\boxed{\omega_{max} = \sqrt{\frac{\mu_s g}{r}}}$

30. MOTION OF A CYCLIST :

A cyclist moving on a circular path of radius r bend away from the vertical by an angle θ. If μ is coefficient of friction, then for no skidding of cycle (or overturning of cyclist)

$$\mu \le \tan\theta = \frac{v^2}{rg}$$

31. VEHICLE ON A LEVEL ROAD

When a vehicle goes around a curve, it has a tendency of skidding side ways i.e. away from the center of the curve. Maximum speed for no skidding, $\upsilon_{max} = \sqrt{\mu_s rg}$

32. CONICAL PENDULUM

The bob is given a horizontal push a little through angular displacement θ and arranged such that the bob describes a horizontal circle with uniform angular velocity ω in such a way that the string always makes an angle θ with the vertical.

$T\cos\theta = Mg$ --- (1)

$T\sin\theta = Mr\omega^2$ --- (2)

$$\tan\theta = \frac{r\omega^2}{g}\ i.e.\ \omega = \sqrt{\frac{g\tan\theta}{r}}$$

33. If a particle is moving along a circle such that its speed is increasing at uniform rate then the particle possesses the following three accelerations

(1) Centripetal acceleration $\left(\vec{a}_r\right)$ (2) Tangential acceleration $\left(\vec{a}_t\right)$ (3) Angular acceleration $\left(\vec{\alpha}\right)$

Resultant linear acceleration $a = \sqrt{a_r^2 + a_t^2}$

- When a particle is moving along a circle at uniform speed, then $\alpha = 0$ and $a_t = 0$.

34.**Relation Between Torque and Angular Momentum**

- Rate of change of angular momentum of a rotating body is equal to the external torque acting on the body

$$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha$$ where

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{t}$$

- $\tau = I\alpha \quad and \quad \tau = F \times d$

where d is perpendicular distance between axis of rotation and line of action of force.

Therefore $I\alpha = F \times d$

35.**Law of conservation of angular momentum:**

Angular momentum of a rotating body remains constant when no external torque acts on it.

i.e. $I\omega = cons\tan t$ when $\tau_{ext} = 0$

(or) $I_1\omega_1 = I_2\omega_2$

36. **MOTION OF A BODY ALONG VERTICAL CIRCLE WITH UNIFORM SPEED:**

A body of mass m is rotated at uniform speed V in a vertical circle of radius r with the help of light string. Let T_1, T_2, T_θ are tensions in the string when the body is at highest point, lowest point of vertical circle and at an angle θ from lowest point of vertical circle respectively

then, $T_2 = \frac{mv^2}{r} + mg$ $\quad T_1 = \frac{mv^2}{r} - mg$

$$T_\theta = \frac{mv^2}{r} + mg\cos\theta$$

$$T_2 - T_1 = 2mg$$

Note :When a body is rotated along vertical circle with uniform speed then sum of K.E and P.E is not constant

37.The expression for maximum speed of vehicle at the highest point of fly-over bridge so that the vehicle does not leave the surface of bridge is

$$\left(\frac{mV^2}{r}\right)_{max} = mg \qquad V_{max} = \sqrt{gr}$$

38.**MOTION OF A BODY ALONG VERTICAL CIRCLE WITH NON UNIFORM SPEED:**

- A body of mass m is suspended from a fixed point with the help of a light string. The body is given a velocity V_2 along horizontal direction so that it describes a vertical circle of radius r. If V_1 is the velocity of body at the highest point of vertical circle, then the sum of K.E and P.E is constant.

$$V_2^2 - V_1^2 = 4gr$$

- If T_2, T_1 are tensions in the string when the body is at lowest point and highest point of vertical circle, then

$$T_2 = \frac{mv_2^2}{r} + mg \qquad T_1 = \frac{mv_1^2}{r} - mg \qquad T_2 - T_1 = 6mg$$

- The body describes vertical circle if minimum $T_1 = 0$; minimum $V_1 = \sqrt{gr}$; minimum $V_2 = \sqrt{5gr}$.

<u>**Note :**</u> V_{min} does not depend on the mass of the body.

T_{min} in the string does not depend on the radius of vertical circle.

- The sphere is made to move along a vertical circle of radius r, during one revolution, then work done by gravity $\Delta K.E = \Delta P.E = 2mgr$

- A small block is freely sliding down from the top of a smooth inclined plane. The block reaches bottom of inclined plane then it describes vertical circle of radius r along a smooth track. If h is minimum vertical height of that inclined plane then $\boxed{h=\frac{5r}{2}}$

39.**Motion of a rolling body :**

- **Total K.E of a rolling body :**

$$K.E_{total}=K.E_{translatory}+K.E_{rotational}=\frac{1}{2}mV^2+\frac{1}{2}I\omega^2=\frac{1}{2}mV^2\left(1+\frac{K^2}{R^2}\right)$$

$$KE_{trans.}:KE_{rot.}:KE_{tot.}=1:\frac{K^2}{R^2}:\left(1+\frac{K^2}{R^2}\right)$$

40. **Rolling of a body on an inclined plane without slipping**

When a body is rolling down on an inclined plane.

- Velocity of the body when it reaches the bottom is given by

$$v=\sqrt{\frac{2gh}{1+\frac{k^2}{r^2}}}=\sqrt{\frac{2gl\sin\theta}{1+\frac{k^2}{r^2}}}\left(\text{since } h=l\sin\theta\right)$$

- Acceleration of the body is given by $a=\dfrac{g\sin\theta}{1+\frac{k^2}{r^2}}$

- Time taken by the body to reach the bottom is given by $t=\sqrt{\dfrac{2l\left(1+k^2/r^2\right)}{g\sin\theta}}$

$$=\sqrt{\frac{2h\left(1+k^2/r^2\right)}{g\sin^2\theta}}\quad\left(\text{since } h=l\sin\theta\right)$$

- The frictional force $(f)=mg\sin\theta\left[\dfrac{k^2}{k^2+r^2}\right]$

- The coefficient of frcition $(\mu)=\tan\theta\left[\dfrac{k^2}{k^2+r^2}\right]$

41 • If two identical spheres each of radius 'r' made up of same material are kept in contact with each other, the gravitational force acting between them

$F\alpha r^4$

- If two identical spheres each of radius 'r' made up of same material are separated by a constant distance then $\Rightarrow F\propto r^6$

- A mass 'm' is split into two parts and separated by certain distance, the gravitational force between them is maximum only when the two parts are of equal mass i.e., $\frac{M}{2}$ and $\frac{M}{2}$.

42.**VARIATION OF 'g' :**

- **Effect of altitude :** If g and g_1 are acceleration due to gravities on the surface of the earth and height 'h' above the surface of the earth of mass M and radius R then hence,

$$\frac{g_1}{g}=\frac{R^2}{(R+h)^2}\Rightarrow g_1=g\left(\frac{R}{R+h}\right)^2$$

- For small values of h, $g_1=g\left(1-\frac{2h}{R}\right)$

- **Effect of depth :** If g is acceleration due to gravity at the surface of the earth and g_1 is acceleration due to gravity at a depth d below the surface of the earth, then $g_1=g\left(1-\frac{d}{R}\right)$

- **Effect of rotation of earth :** Due to the rotation of earth, the value of acceleration due to gravity g^1 at a given place is given by

$g^1=g-r\omega^2\cos\lambda$

- At the poles $\lambda=90^0\ (\cos 90=0)\ \therefore g^1=g$
i.e. The value of g is maximum at poles

- At the equator $\lambda=0^0\ (\cos 0=1)$
$\therefore g^1=g-\omega^2R$ i.e., The value of g is minimum at equator.

- The time period of earth

$$T=2\pi\sqrt{\frac{R}{g}}=84.6\,\text{min}=5076s$$

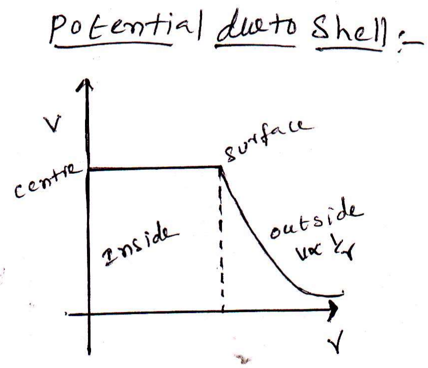
43.• **NULL CONCEPT :** The point between two massive object at which a fields then are equal in magnitude but opposite in direction is called null point. Null distance equals to

$$x=\frac{d}{\sqrt{\frac{m_2}{m_1}}+1}$$

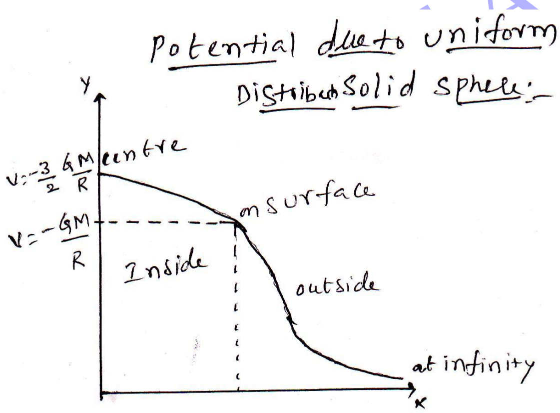
(x is the distance all ways from smaller mass)

44.**GRAVITATIONAL POTENTIAL:** Gravitational potential mass "M" $V = -\frac{GM}{R}$ (M is mass of the massive object)

- **Gravitational potential due to a spherical shell:-** mass of shell is M,R is radious then
  At a point inside the spherical shell,



- **Gravitational potential due to a solid sphere:**
  At a point inside the solid sphere,



45. **GRAVITAITONAL POTENTIAL ENERGY:**

- The work is done in bringing an object of mass m from ∞ to a point in the gravitational field of massive object. $\Delta U = \frac{-GMm}{R}$

- Change in gravitational potential energy in lifting an object from the surface of a planet to an altitude h is given by

$$\Delta U = \frac{GMm}{R\left[1+\frac{R}{h}\right]} = \frac{mgh}{1+\frac{h}{R}}$$

46.**ESCAPE VELOCITY:**

- $\boxed{V_e = \sqrt{\frac{2GM}{R}}} \Rightarrow \boxed{V_e \infty \sqrt{M}}$ (if Radius, R is constant)

  $\Rightarrow \boxed{V_e = \sqrt{2gR}}$ $\left[\because g = \frac{GM}{R^2}\right]$

  $\Rightarrow \boxed{V_e = R\sqrt{\frac{8\pi G\phi}{3}}} \Rightarrow V_e \infty R$ (if density of the earth planet is constant)

or

Salient features regarding escape velocity:

- Escape velocity depends on the mass, density and radius of the planet from which the body is projected.
- Escape velocity does not depend on the mass of the body, its direction of projection and the angle of projection.
- the body will move in interplanetary or interstellar space with a velocity $\sqrt{V^2 - V_e^2}$

47.**Kepler's second law or <u>LAW OF AREAS</u>:**

- $\frac{L}{2m}$ = constant, this law obeys law of conservation of angular momentum, it is also called "Law of Areas".

- A real velocity $\frac{dA}{dt}$ =constant,i.e., Kepler's II law or constancy of a real velocity is a consequence of conservation of angular momentum.According to the law of conservation of angular momentum

  $m(V_{\max})(r_{\min}) = m(V_{\min})(r_{\max})$

49.• **ORBITAL VELOCITY:**

- $V_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$

50.**TIME PERIOD OF REVOLUTION:**

Time taken by the satellite in completing one revolution round the earth is called as its time period (T).Time period of revolution,

$$T = \frac{circumference of the orbit}{orbital velocity} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

51.**<u>Binding energy</u> (E_B) :**

The minimum energy required by the satellite in order to make it escape from the gravitational field of a planet is defined as the binding energy

E_b = -Total energy = $\frac{GMm}{2r} = \frac{GMm}{2(R+h)}$

52. **ANGULAR MOMENTUM:**

- In case of the satellite motion, the angular momentum of the satellite is given by

  $\Rightarrow L = \sqrt{GMm^2 r}$

  force is central ,so for satellites τ =0, then L=constant
- Angular momentum of the satellite depends on mass of the satellite, mass of the planet and radius of the orbit.

53.— **Velocity :**

- The rate of change of displacement is velocity. $v = \frac{dy}{dt} = A\omega\cos\omega t = \omega\sqrt{A^2 - y^2}$
- **Average speed in SHM:**
- The average speed during one complete oscillation is given by $V_{Avg} = \frac{4A}{T}$, where T is time period of oscillation.

  Note: Average velocity during one complete oscillation is zero.

- A particle is vibrating in SHM. If its velocities are V_1 and V_2 when the displacements from the mean position are y_1 and y_2 respectively. then its time period is $2\pi\sqrt{\frac{y_2^2 - y_1^2}{V_1^2 - V_2^2}}$
- Phase difference between displacement and velocity of S.H.O. = $\frac{\pi}{2}$ radian = 90 °
- Phase difference between displacement and acceleration of S.H.O. = π radian = 180 °
- Phase difference between velocity and acceleration of S.H.O. = $\frac{3\pi}{2}$ or $\frac{\pi}{2}$ radian
- Equation of motion of a simple pendulum in differential form is $\frac{d^2y}{dt^2} + \omega^2 y = 0$, where

  $\omega^2 = g/l$
- The time period of a simple harmonic oscillator is $T = 2\pi\sqrt{\frac{displacement\,(y)}{acceleration\,(a)}}$

54. **Simple Pendulum :**

- Time period of a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

  - $T \,\alpha \sqrt{l}$ or $\frac{l}{T^2}$ = constant (at a place)
- $\frac{l_1}{l_2} = \frac{T_1^2}{T_2^2}$ • for smaller percentages, $\frac{\Delta T}{T}\% = \frac{1}{2}\frac{\Delta l}{l}\%$
- $T \,\alpha \frac{1}{\sqrt{g}}$ or gT^2 = constant (if length of the pendulum is constant) • $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$
- l - T^2 graph of a simple pendulum is **straight line** passing through origin.
- l -T graph of a simple pendulum is **parabola.**
- When the elevator is going up with an acceleration a, then its time period is given by

  $T = 2\pi\sqrt{\frac{L}{g+a}}$

  and frequency n is given by $n = \frac{1}{2\pi}\sqrt{\frac{g+a}{L}}$
- When the elevator is moving down with an acceleration a, then its time period is given by

  $T = 2\pi\sqrt{\frac{L}{g-a}}$ and frequency n is given by

  $n = \frac{1}{2\pi}\sqrt{\frac{g-a}{L}}$
- If a simple pendulum of length L is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination 'θ'. Then the time period of oscillation is given by

- A simple pendulum fitted with a metallic bob of density d has a time period T. When it is made to oscillate in a liquid of density d_L , then itstime period increases.

$$T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{d_l}{d_s}\right)}} = 2\pi\sqrt{\frac{l}{g\left[1-\frac{1}{d_{rel}}\right]}}$$

- When two simple pendula of lengths l_1 *and* l_2 are set into vibration in the same direction at the same instant with same phase, again they will be in same phase after the shorter pendulum has completed n oscillations. To find the value of n. $n\ T_s = (n-1)T_L$ *and* $T\ \alpha \sqrt{l}$

$$\therefore \frac{n}{n-1} = \frac{T_L}{T_s} \quad or \quad \frac{n}{n-1} = \sqrt{\frac{l_L}{l_S}},$$

L= longer, S=shorter

55.Spring - mass system :

- Resorting force $F = -kx$;
- When the load is pulled down and released, it makes vertical oscillations $T = 2\pi\sqrt{\frac{M}{k}}$
- Time period of spring mass system is independent of acceleration due to gravity.
- When a clock fitted with spring mass system is taken to the surface of the moon, it's time period will remain constant.
- When a spring of force constant k and length l is cut into two parts of lengths l_1 and l_2 having force constants k_1 and k_2

$$k\ \alpha\ \frac{1}{l}\ \ or\ \ kl = k_1 l_1 = k_2 l_2$$

- If a spring of spring constant 'k' is divided into 'n' equal parts, then spring constant of each part is 'nk'.
- If a spring of spring constant 'k' and length 'l' is cut into two springs of lengths 'l_1' and 'l_2' then the spring constants of the two parts are

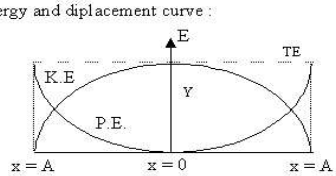
$$k_1 = \frac{k(l_1+l_2)}{l_1} \quad and \quad k_2 = \frac{k(l_1+l_2)}{l_2}$$

- When two springs of force constants k_1 and k_2 are connected in series, then the effective force constant is $k = \frac{k_1 k_2}{k_1 + k_2}$
- When two springs of force constants k_1 and k_2 connected in parallel, the effective spring constant is $k=k_1+k_2$

56. **Energy of simple harmonic oscillator :**

- **Potential Energy :** P.E. = $\frac{1}{2}m\omega^2 x^2$ = $2\pi^2 m\nu^2 x^2$ Where ν - frequency of oscillation
- **Kinetic Energy :** — K.E. = $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$
- T.E. is constant at all positions of S.H.O.,

Energy and diplacement curve :



- Displacement and acceleration graph of a S.H.O. is a straight line passing through origin.
- Displacement and velocity graph of a S.H.O. is an ellipse.
- If 'n' is the frequency of oscillation of a S.H.M , then its P.E and K.E veries with a frequency of 2n.

57. **Stress :**

- The restoring force per unit area is called stress.

$$\text{Stress} = \frac{\text{restoring force}}{\text{area of cross section}} = \frac{F}{A}$$

- **Young's modulus (Y) :**

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F\ell}{Ae}$$

; If load attached to the wire is M, then F = Mg, and $A = \pi r^2$

$$Y = \frac{Mg\ell}{\pi r^2 e}$$

- If the load attached to the wire and Y are constant $e \propto \frac{\ell}{r^2}$; If length is also constant $e \propto \frac{1}{r^2}$

If volume of the wire is constant

$$e \propto \ell^2 \; ; \; e \propto \frac{1}{A^2} \; ; \; e \propto \frac{1}{r^4}$$

- Elongation of a wire under its own weight

$$e = \frac{\ell^2 dg}{2Y}$$ (d is density).

- If ℓ_1, ℓ_2 are the lengths of a wire under tensions $T_1 \& T_2$ then the length of the unstretched wire.

$$\ell = \frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$$

- When a load is suspended from a wire its elongation is e. If the load is immersed in a liquid of density 'ρ' then the new elongation. $e^1 = e(1 - \rho / d)$ d is density of load
- For a perfectly elastic material e = 0. So Young's modulus is infinity.
- For a plastic material the Young's modulus is zero.
- Two wires of same length and radius are joined end to end and loaded. If Young's moduli of the materials are $Y_1 \& Y_2$, the Young's modulus of the combination is $\frac{2}{Y} = \frac{1}{Y_1} + \frac{1}{Y_2}$;

$$Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

58. **Bulk modulus (K)**

With in the elastic limit the ratio between volume stress and bulk strain is called bulk modulus.

Bulk modulus $= \frac{\text{volume stress}}{\text{bulk strain}}$

$$K = \frac{\frac{F}{A}}{\frac{-\Delta V}{V}} = -\frac{V}{\Delta V}\frac{F}{A} = \frac{-PV}{\Delta V}$$

- If a block of coefficient of cubical expansin γ is heated through a rise in temperature of θ, the pressure to be applied on it to prevent its expansion = $K\gamma\theta$, where K is its bulk modu lus.

59 **Poisson's ratio (σ)**

- The ratio of lateral contraction strain to the longitudinal elongation strain is called Poisson's ratio.

$$\sigma = \frac{\text{lateral contraction strain}}{\text{longitudinal elongation strain}}$$

$$= \frac{\text{transverse strain}}{\text{longitudinal strain}} = \frac{\Delta r / r}{\Delta \ell / \ell} = \frac{\Delta r \ell}{\Delta \ell \text{ x } r}$$

- As it is a ratio, it has no units and dimension.
- Theoretical limits of $\sigma = -1 \text{ to } 0.5$
- Practical limits of $\sigma = 0 \text{ to } 0.5$
- For an incompressible substance $\sigma = 0.5$

60. **Breaking Stress :**

- The stress required to break the wire is called breaking stress. =

$$\frac{\text{breaking force}}{\text{initial area of cross section}}$$

$$= \frac{Mg}{A} = \frac{A\ell dg}{A} = \ell dg$$

- The maximum length of the wire that can hangwithout breaking under its own weight

$$\ell = \frac{\text{breaking stress}}{dg}$$

- Breaking stress depends on the nature of the material, but it is independent of dimensions.
- Substances like tissue of aorta, rubber etc.,which can be stretched to cause large strains are called elastomers.

61. **Thermal force :**

- When a metal bar is fixed between two supports and heated, it tries to expand and exerts force on the walls. This is called thermal force.

$$F = AY\alpha\theta$$

- Thermal force is independent of length of the bar. Thermal stress (linear compressive stress)

$$\frac{\text{force}}{\text{area}} = \frac{AY\alpha\theta}{A} = Y\alpha\theta.$$

62 **Spring**

- For a given spring $F \propto x$; $F = kx$; $k = \frac{F}{x}$

  k is called spring constant (or) force constant (or) stiffness constant.

- Spring constant interms of Young's modulus area of cross section and length $k = \frac{yA}{\ell}$.

- P.E.energy of a streched spring

  $$E = \frac{1}{2}Kx^2 \Rightarrow \frac{1}{2}Fx = \frac{F^2}{2K}$$

- Two springs having force constants $K_1 \& K_2$ $(K_1 > K_2)$ are stretched by same amount then more work is done on the first spring $W \propto K$.

- Two springs having force constants $K_1, K_2 (K_1 > K_2)$ are stretched by same force then more work is done on the second spring. $W \propto 1/K$.

- If energy is same for both the springs the relation between force and spring constant $F \propto \sqrt{K}$.

- The work done in stretching a wire

  $W = \frac{1}{2}$ Stress x Strain x Volume;

  $$W = \frac{1}{2}Fe = \frac{1}{2}\frac{e^2 AY}{\ell}$$

- Strain energy per unit volume (energy density)

  $$E = \frac{1}{2} \text{x stress x strain} = \frac{1}{2}Y(Strain)^2$$

  Also, $E = \frac{(Stress)^2}{2Y} \left[\therefore Y = \frac{Stress}{Strain} \right]$

63.**FORCE DUE TO SURFACE TENSION:**

- Force required to pull a wire of length 'l' from the surface of water of surface tension T is

  F=2lT

64.SURFACE ENERGY:

- Work done in forming a liquid drop is,
  w = Surface Area x Surface tension
  = AT = 4 π r² T.

- Work done in increasing the size of liquid drop from radius r_1 to r_2 is W = 4 π T(r_2^2 - r_1^2)

- Work done in blowing a soap bubble is
  w = 8 π r² T.

- Work done in increasing the size of a soap bubble from radius r_1 to r_2 is, W = 8 π T (r_2^2 - r_1^2)

- Work done in forming a circular liquid film of radius 'r' is, w = 2 π r² T.

- Work done in increasing the area of circular soap film from radius r_1 to r_2 is

  W = 2 π T (r_2^2 - r_1^2).

- When a liquid drop of radius 'R' splits into 'n' identical droplets, then the total surface area of 'n' droplets will be greater than that of big drop. So energy is absorbed.

- Work done to split a liquid drop of radius 'R' into 'n' no. of identical droplets is

  W = 4 π R² T ($n^{1/3}$ - 1).

- When 'n' identical droplets are combined to form a big drop, then the energy will be released.

- Work done to form a big drop from 'n' identical droplets each of radius 'r' is W = 4 π r² T (n - $n^{2/3}$).

65.CAPILLARITY:

- The weight of the liquid column in the capillary tube is balanced by the force due to surface Tension.

  2 π r T cos θ = Mg

- Surface Tension by capillary tube method
  If r is very small compared to h, then

  $T = \frac{hrdg}{2\cos\theta}$ where r = radius of the capillary tube
  h = height of liquid column d = density of the liquid
  g = acceleration due to gravity θ = angle of contact

- When diameter of capillary tube increases twice, the height of liquid column falls down to half. ($r_1h_1 = r_2h_2$).

- When capillary tube is dipped vertically in a liquid then the rise in the liquid is h. If the tube is tilted by making the inclination 'θ' with vertical then the slant height of the liquid is l = h/cos θ.

- A capillary tube is vertically dipped in a liquid. The height of the liquid in the tube is 'h' and the total set up is kept in a lift.
- If the lift is moving up with an acceleration 'a' then the height of the liquid in the tube is given by $h' = h\left[\frac{g}{g+a}\right]$
- If the lift is moving down with an acceleration 'a' then the height of the liquid in the tube is given by

$$h' = h\left[\frac{g}{g-a}\right]$$

66.EXCESS PRESSURE INSIDE A LIQUID DROP & SOAP BUBBLE

- Excess pressure inside a liquid drop of radius r is given by P = 2T/r.
- Excess pressure in a soap bubble of radius r is given by P = 4T/r.
- When two soap bubbles of radii a and b in vaccum coalesce under isothermal condition, then radius of the resultant bubble is,

$$R = \sqrt{a^2 + b^2}$$

- When a soap bubble of radius r_1 and another of radius r_2 are brought together, then the radius of curvature 'R' on the common interface is

$$R = \frac{r_1 r_2}{r_2 - r_1}\left(If\ r_2 > r_1\right)$$

67. EQUATION OF CONTINUITY :

- When an incompressible fluid flows through a tube of non-uniform cross section, the product of area of cross section and velocity of flow is constant.
  A $\times$ v = Constant $\Rightarrow A_1 v_1 = A_2 v_2$
- In the case of horizontal pipe in which liquid flows,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

Where P_1 and P_2 are pressures at two points, v_1, v_2 are velocities at two points and ρ is the density of the liquid.

68. TORRICELLI'S THEOREM:

- The velocity of efflux of a liquid through an orifice is equal to that of the velocity acquired by a freely falling body from a height which is equal to that of the liquid level from the orifice. $v = \sqrt{2gh}$
- Time taken by the efflux liquid to reach the ground is given by $t = \sqrt{\frac{2(H-h)}{g}}$
- Horizontal range of the liquid is given by R = $V \times t$

$$R = \sqrt{2gh.\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$$

- Horizontal range is <u>maximum</u> when the orifice is at the middle of liquid level and bottom.

i.e., if $h = \frac{H}{2}$ then $R_{Max} = H \Rightarrow 2h$

- A cylindrical vessel of area of cross section 'A' has a hole of area of cross section 'a' at its bottom. Time taken for the water level to decrease from h_1 to h_2 as water flows out from the hole is

$$t = \frac{A}{a}.\sqrt{\frac{2}{g}}\left[\sqrt{h_1} - \sqrt{h_2}\right]$$

- The volume of the liquid coming out of the orifice per second = $V = A.\sqrt{2gh}$ = $\pi r^2.\sqrt{2gh}$ ($since V = Av$)
- Dynamic lift = $(P_2 - P_1)A = \frac{1}{2}\rho.(V_1^2 - V_2^2) \times A$

69. VISCOUS FORCE:(NEWTON'S FORMULA)

- The viscous force acting between two adjacent layers of a liquid is directly proportional to the surface area of the layers in contact and the velocity gradient. $F = -\eta A \frac{dv}{dx}$

STOKE'S FORMULA :

- The viscous force acting on a spherical body of radius 'r' moving with velocity v in a liquid of coefficient of viscosity 'η' is given by

  $F = 6\pi\eta r v$
- For a spherical body is dropped in a fluid, when the viscous force balances the apparent weight of the body, it travels with uniform velocity and this is called terminal velocity (V_T).

$$V_T = \frac{2}{9}.\frac{gr^2[d-\rho]}{\eta}$$

70.SAME EXPANSION IN DIFFERENT RODS

→ If two rods of different materials have the same deference between their lengths at all temperatures.

Then $l_1\alpha_1 = l_2\alpha_2$, $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$ if the constant deference in their lengths is x then $l_1 = \frac{x\alpha_2}{\alpha_1 \sim \alpha_2}$,

$$l_2 = \frac{x\alpha_1}{\alpha_1 \sim \alpha_2}$$

PENDULUM CLOCKS:

→ Pendulum clocks lose or gain time as the length increases or decreases respectively.

→ The fractional change in time period =

$$\frac{\Delta T}{T} = \frac{\alpha\Delta t}{2}$$

→ The loss or gain per day = $\frac{1}{2}\alpha\Delta t \times 86400$ Seconds.

→ % Time lost (or) gained = $\frac{1}{2}\alpha\Delta t \times 100$

71. **VARIATIONOF DENSITY OF A SOLID WITH TEMPERATURE.**

→ $d_t = \frac{d_0}{(1+\gamma t)}$ (or) $d_t = d_o(1+\gamma t)^{-1}$ (exact formula)

72. • **RELATION BETWEEN γ_r AND γ_a :**

→ The coefficient of real expansion of a liquid is equal to the sum of coefficient of apparent expansion of the liquid and coefficient of volume expansion of the vessel.

$$\gamma_r = \gamma_{app} + \gamma_{vessel} = \gamma_{app} + 3\alpha_{vessel}$$

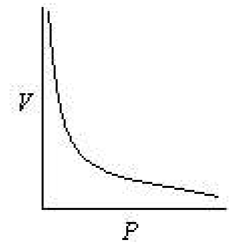
73. **VOLUME OF UNOCCUPIED SPACE REMAINS SAME AT ALL TEMPERATURES:**

→ When a liquid is taken in a container and heated, the unoccupied volume over the liquid remains constant at all temperatures, if $V_c\ \gamma_c = V_l\ \gamma_l$

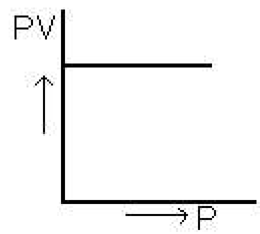
→ **ANAMOLOUS EXPANSION OF WATER:**
At 4^0 C water occupies minimum volume and hence density becomes maximum (1gm /cc).

74.

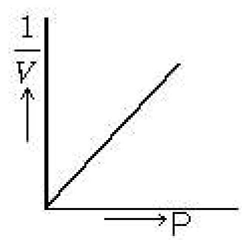
→ P-V graph at constant temperature is a rectangular hyperbola.



- PV-P graph is a horizontal straight line parallel to pressure axis.



- P-1/v graph is a straight line passing through origin



- Area of P-V Graph drawn on to volume axis gives amount of work done by the gas
- Two vesseles of volumes V_1 and V_2 containing a gas under pressures P_1 and P_2 respectively are joined at the same temperature. Then the common pressure $P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$ If second vessel is an evacuated one then

$$P = \frac{P_1V_1}{V_1 + V_2}$$

- If an air bubble raises from the bottom to surface of a lake,if atm pressure is $H\ m$ of water.if the volume of the bubble becomes n times to its initial volume then the depth of the lake $h = H(n-1)$
If the radius of bubble becomes n times at constant temperature on reaching the surface then $h = H(n^3 - 1)$
- **NUMBER OF MOLECULES PER UNIT VOLUME:**

$$PV = nRT = (n\,N)\frac{R}{N}.T \;or\; P = \left(\frac{nN}{V}\right)kT$$

So number of molecules per unit volume = $\frac{P}{kT}$

- If a gas with physical parameters (P_1,V_1,T_1) is mixed with another gas at (P_2,V_2,T_2) then the resultant mixture is at (P,V,T) then

$$\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2} = \frac{PV}{T}$$

- **Root mean square speed (r.m.s.)**

$$: V_{rms} = \sqrt{\frac{3PV}{mass\ of\ gas}}$$

$$= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m}}$$

($\because$ R= NK and $M = Nm$)

where M is the molecular weight while *m* the mass of a single molecule.

- **Most probable speed :**
The speed possessed by maximum number of molecules in a gas at constant temperature is called most probable speed.

$$V_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2}{3}}\,V_{rms} = 0.816\ V_{rms}$$

- **Average speed :**
It is the arithmetic mean of the speeds of molecules in a gas at a given temperature.

i.e. $V_{av} = \frac{(V_1 + V_2 + ... + V_n)}{n}$ According to kinetic theory of gases

$$V_{av} = \sqrt{\frac{8RT}{\pi m}} = \left(\sqrt{\frac{8}{3\pi}}\right)V_{rms} = 0.92\,V_{rms}$$

- $V_{rms} > V_{av} > V_{mp}$

- A planet or satellite will have atmosphere only if $V_{rms} < V_e$, where V_e is escape velocity of planet or satellite.

75. **Some important points of kinetic theory of gases :**

- Pressure P exerted by an ideal gas is given by

$$P = \frac{1}{3}\frac{mN}{V}\left(\overline{V}\right)^2 = \frac{2}{3}E$$

- Mean kinetic energy per gram mole of a gas is given by

$$E_{mol} = \frac{3}{2}KTN = \frac{3}{2}RT$$

- Average translation K.E of a gas molecule depends only on its temperature and is independent of its nature.
- The average distance travelled by a molecule between two successive collisions is called 'mean free path' and is given by $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$
where 'd' is the diameter of molecules and *n* is the number of molecules per unit volume.

76.♦ **Degree of freedom :**

- **Relation between γ and f :**

$$\boxed{\gamma = 1 + \frac{2}{f}} \quad \text{where } \gamma = \frac{C_p}{C_V}$$

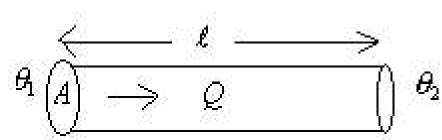
- The internal energy of *n* moles of a gas in which each molecule has *f* degrees of freedom, will be

$$U = \frac{1}{2}nfRT$$

- for monoatomic gas, $f = 3$, so $U = \frac{3}{2}nRT$
- for diatomic gas, f = 5, so $U = \frac{5}{2}nRT$
- for non linear polyatomic gas, f = 6 so $U = \frac{6}{2}nRT$

77 **Coefficient of Thermal conductivity :**

- In steady state the quantity of heat Q flowing through a metal rod of length ℓ and cross-section A in a time t when its ends are at temperature θ_1 and θ_2 $(\theta_1 > \theta_2)$ is given by



$$Q = \frac{KA(\theta_1 - \theta_2)}{l}t$$

where K is coefficient of thermal conductivity

- K depends on the nature of the metal.
- It is defined as the rate of flow of heat per unit

area and per unit temperature gradient in steady state.

- Units of K : S.I - $Wm^{-1}K^{-1}$ $C.G.S - cals^{-1}cm^{-1}C^{-1}$
- Dimensional formula - $\left[MLT^{-3}K^{-1}\right]$
- Values of K :
- For a perfect conductor $K = \infty$.
- For a perfect insulator K= 0
- **Junction temperature :** In steady state when conduction takes place through two layers of composite wall with different thermal conductivities, then $\theta = \dfrac{\dfrac{K_1\theta_1}{l_1} + \dfrac{K_2\theta_2}{l_2}}{\dfrac{K_1}{l_1} + \dfrac{K_2}{l_2}}$
- **Thermal Resistance : R**
  Thermal resistance R of a conductor of length ℓ, cross-section A and conductivity K is given by

$R = \dfrac{l}{KA}$

- S.I unit :- KW^{-1}
- Dimensional formula : $\left[M^{-1}L^{-2}T^{3}K\right]$

Effective conductivity :

- **Series combination :**

$$K = \frac{K_1K_2\left(l_1 + l_2\right)}{K_1l_2 + K_2l_1}$$

- **Parallel combination :**

$$K = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

78. **GROWTH OF ICE ON PONDS**

When the temperature of air over the lakes fall below 0^0C, ice is formed and floats on the water. The thickness of thin layer of ice gradually increases. The time taken by ice to increase its thickness from x_1 to x_2 is

$$t = \frac{\rho L}{2k\theta}\left(x_2^2 - x_1^2\right)$$

where L=Latent heat of fusion of ice ρ =Density of ice

79. **CONVECTION:-**

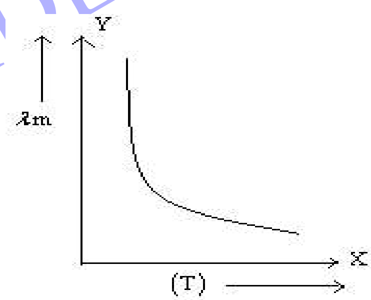
- Natural convection can not take place in a gravity free space such as orbiting satellite or freely falling lift.
  EX: Ventilators placed below the roof allow hot air to escape.

80. **RADIATION:**

- It is the process of transmission of heat from one place to another without any material medium.
- Every object emits and absorbs radiant energy at all temperatures except at absolute zero.
- **WIEN'S LAW:**
- According to this law. the wavelength $\left(\lambda_m\right)$ corresponds to which energy emitted per sec per unit area by a perfectly body is maximum, is inversely proportional to the absolute temp(T) of the black body.

$\lambda_m \propto \dfrac{1}{T}$ or $\lambda_m T = b$ where 'b' is known as wien's constant $b = 2.9 \times 10^{-3}$ mk

This law has been used for determining temperatures of distant stars. The variation of λ_m versus T is as shown below



- **STEFAN'S LAW:**
- Dimensional formula of Stefan's constant is $MT^{-3}\theta^{-4}$.
- Radiant energy emitted by a hot body per second = $eA\sigma T^4$ where e is the emissivity of the hot body, A its surface area, T its absolute temperature and σ the Stefan's constant. (for perfect black body, e = 1)
- σ = Stefan's constant = 5.67 $\times 10^{-8} Wm^{-2}K^{-4}$
- **STEFAN - BOLTZMAN'S LAW:**

If a black body at absolute temperature T is surrounded by an enclosure at absolute temperature T_o, then the rate of loss of heat energy by radiation per unit area is given by.

$$E = \sigma(T^4 - T_0^{\,4})$$

- For any hot body, $E = \sigma Ae(T^4 - T_0^4)$

Where 'e'is the emissive power and 'A' is the area of cross-section of the hot body.

- **NEWTON'S LAW OF COOLING:**
  - The rate of cooling of a hot body is directly proportional to the mean excess of temperature of the body above that of the surroundings, provided the difference in temperature of the body and that of surroundings is small.

$$\frac{d\theta}{dt} = K\left(\frac{\theta_1+\theta_2}{2} - \theta_s\right).$$

or $(\theta-\theta_0)_1 = (\theta-\theta_0)_2$

- When a solid sphere of radius R, density D and specific heat S is heated to temperature θ and then cooled in an enclosure to temperature θ_0, then its rate of fall of temperature $\frac{d\theta}{dt} \propto \frac{1}{RDS}$

81.APPLICATIONS OF JOULE'S LAW

- The height from which ice is to be dropped to melt it completely is

$h = \frac{JL}{g}$ where L= Latent heat of ice.

- The rise in temperature of water when it falls from a height h to the ground is,

$$\Delta\theta = \frac{gh}{Js}$$

where 's' is specific heat of water

- When a body of mass m moving with a velocity v is stopped and all of its energy is retained by it , then the increase in temperature is. $\Delta\theta = \frac{v^2}{2Js}$

- **FIRST LAW OF THERMODYNAMICS:**
  - All the heat added to a system is partly utilised to do the mechanical work and remaining to increase its internal energy.
    - The differential form of first law of thermodynamics is

    dQ=dU+dW, where

    dQ = heat added,

    dU = Increase in internal energy.

    dW= work done
  - It defines the property of system called internal energy.
  - It is a consequence of law of conservation of energy.

- **SIGN CONVENTIONS:**
  - When heat is added to the system dQ is +ve(+dQ)
  - When heat is taken from the system dQ is -ve (-dQ)
  - When gas expands work is done by the gas dw is positive (+ dW)
  - When gas contaracts work is done on the gas dw is negative.(-dW)
  - When internal energy of system increases and dU is +ve (+dU)
  - When internal energy of system decreases and dU is -ve(-dU)

- **HEAT CAPACITY(OR) THERMAL CAPACITY:**
  - It is the amount of heat required to raise the temperature of the body by 1^0C

$$c = \frac{\Delta Q}{\Delta T} = MS$$

  - The S.I.unit is JK^{-1}
  - The C.G.S unit is Cal $^oC^{-1}$
  - Dimensional formula : $ML^2T^{-2}K^{-1}$
  - Thermal capacity depends on mass and nature of substance.

- **LAW OF MIXTURES (OR) CALORIMETRY PRINICPLE:**
  - When three substances of different masses m_1, m_2 and m_3 specific heats s_1, s_2, s_3 and at different temperatures t_1,t_2, and t_3 respectively are mixed, then the resultant temperature is

$$t = \frac{m_1s_1t_1 + m_2s_2t_2 + m_3s_3t_3}{m_1s_1 + m_2s_2 + m_3s_3}$$

  - When ice at 0^0C and steam at 100^0C are mixed, mass of ice that melts is

    $m_{ice} = 8\ m_{steam}$

    When "x" gram of steam is mixed with "y" gram of ice, the resultant temperature is

$$t = \frac{80(8x-y)}{(x+y)}$$

  - When ice and water are mixed the mass of ice that just melts is

$$m_{ice} = \frac{ms\theta}{80}$$

    where m= mass of water.
  - When ice and steam are mixed, the amount of steam that just condenses into water at 100^0C.

$$m_{steam} = \frac{m_{ice}}{3}$$

- **Cv, Cp AND VALUES OF DIFFERENT GASES:**

S.No.	Nature of gas	C_p	C_v	$\gamma = \frac{c_p}{c_v}$
1.	Monoatomic	$\frac{5}{2}R$	$\frac{3}{2}R$	$\frac{5}{3} = 1.67$
2.	Diatomic	$\frac{7}{2}R$	$\frac{5}{2}R$	$\frac{7}{5} = 1.4$
3.	Tri (or) Polyatomic	4R	3R	$\frac{4}{3} = 1.33$

γ value is always greater than one.It depends upon the atomicity of a gas. It decreases with increase in atomicity.

$$C_p = \gamma \frac{R}{\gamma - 1} \; and \quad C_v = \frac{R}{\gamma - 1}$$

- **γ OF MIXTURE OF GASES:** When n_1 moles of a gas with specific heat at constant volume C_{V1} is mixed with n_2 moles of another gas of specific heat at constant volume C_{v2} then

$$(C_v)_{mixture} = \frac{n_1 c_{v1} + n_2 c_{v2}}{n_1 + n_2}$$

$$(C_p)_{mixture} = (C_v)_{mixture} + R$$

$$\gamma_{mixture} = \frac{C_{p(mixture)}}{C_{v(mixture)}}$$

At constant pressure

- Fraction of heat absorbed that is converted into internal energy is $\frac{dU}{dQ} = \frac{C_v}{C_p} = \frac{1}{\gamma}$
- Fraction of heat absorbed that is converted into workdone= $\frac{dW}{dQ} = \frac{R}{C_p} = 1 - \frac{1}{\gamma}$

82. ISOTHERMAL PROCESS:

- In this process, the pressure and volume of gas changes but temperature remains constant.
- The system is in thermal equilibrium with the surroundings.
- It is a slow process.
- The internal energy of the system remains constant i.e., du=0
- It obeys the Boyle's law i.e. PV=k
- The isothermal elasticity

$$= -\frac{dp}{dv/v} = p \;.$$

The -ve sign represents, as pressure increases volume decreases.

- It takes place in a conducting vessel.

- **ADIABATIC PROCESS:**
  - The pressure, volume and temperature of a gas change but total heat remains constant i.e., dQ=0 (Q=constant).
  - It is a quick process.
  - The internal energy changes as temperature changes.
  - The adiabatic process is represented by the equations.

PV^{γ} = constant $\quad TV^{\gamma-1}$ constant

$P^{1-\gamma}T^{\gamma}$ =constant

83. WORK DONE IN ISOTHERMAL AND ADIABATIC PROCESSES:

- The workdone during the isothermal expansion at constant temperature is

$$W = 2.303RT\log_{10}\left(\frac{V_2}{V_1}\right) = 2.303 RT\log_{10}\left(\frac{p_1}{p_2}\right)$$

- The workdone by the system during the adiabatic expansion is

$$w = \frac{nR}{\gamma - 1}(T_1 - T_2) = nC_v(T_1 - T_2)$$

$$= n\frac{c_p}{\gamma}(T_1 - T_2) \qquad = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

- The adiabatic elasticity of gas is γ p.
- The slope of adiabatic curve is γ times greater than the isothermal curve.

- **DIFFERENT TYPES OF THERMODYNAMIC PROCESSES:**

Sl. No.	Name of the process	Quantity remains constant	Quantity which becomes Zero	Result in I law
1.	Isother-mal	Tempera-ture	dU	dQ=dW
2.	Isobaric	Pressue	None	dQ=du+dW

3.	Isochoric	volume	dW	dQ=dU
4.	Adiabatic	heat	dQ	dU=-dW

- **Efficiency of heat engine -**

i) Efficiency of heat engine (η) is defined as the fraction of total heat, supplied to the engine which is converted into work.
ii) Mathematically -

$$\therefore \eta = \frac{W}{Q_1} \quad \text{or} \quad \therefore \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

we can show that $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

- **REFRIGERATORS:**
- Coefficient of performance of a refrigerator

$$(\beta) = \frac{Q_2}{W} \quad \Rightarrow \beta = \frac{Q_2}{Q_1 - Q_2} \quad [\because W = Q_1 - Q_2]$$

83.In more general form

If $Z = \frac{A^p B^q}{C^r}$ then maximum fractional error in Z is $\frac{\Delta Z}{Z} = P\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

Maximum Percentage error in Z =

$$\frac{\Delta Z}{Z} \times 100 = P\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

- **Significant Figures :**
- **Rules for determining the number of significant figures:**

➢ All the non-zero digits in a given number are significant without any regard to the location of the decimal point if any. Ex: 184,52 has five significant digits.
1845.2 or 184.52 all have the same number of significant digits that is 5.

➢ All zeros occruing between two non zero digits are significant without any regard to the location of deciman point if any. Ex: 106008 has six significant digits.
106.008 or 1.06008 has also got six sifnificant digits.

➢ If the number is less than one, all the zeros to the right of the decimal point but to the first non-zero digit are not significant.
Ex: 0.000308 has 3 significant digits.

➢ a) All zeros to the right of a demcimal point are significant if they are not followed by a non-zero digit.
Ex: 30.00 has 4 significant digits
b) All zeros to the right of the last non-zero digit after the decimal point are significant.
Ex: 0.05600 has 4 significant digits

➢ All zeros to the right of the last non-zero digit in a number having no decimal point are not significant.
Ex: 2030 has 3 significant digits

- **Rules for Arthemetic Operations with significant Figures:**

➢ In multiplication or division, the final result should retain only that many significant figures as are there in the original number with the least number of significant figures.
Ex: $1.2 \times 2.54 \times 3.26 = 9.93648$. But the result should be limited to the least number of significant digits-that is two digits only. So ginal answer is 9.9.

➢ In addition or substraction the final result should retain only that many decimal places as are there in the number with the least decimal places.
Ex: 2.2+4.08+3.12+6.38=15.78. Finally we should have only one decimal place and hence 15.78 is to be rounded off as 15.8.

84. If θ is angle of friction and μ is the coefficient of friction then μ = Tanθ

- When a body is moving on a rough inclined plane which makes an angle θ with the horizontal the frictional force acting on it is

$$F = \mu_k mg \cos\theta$$

MOTION ON A HORIZONTAL ROUGH SURFACE

- When a body of mass m on a horizontal rough surface is pulled or pushed with a horizontal force F,
- If the body does not move, the frictional force is equal to the applied force in backward direction.
- If the body is about to move. Then the Friction force = $\mu_s R = \mu_s mg$
- If the applied force F > $\mu_s mg$, the body moves forward. Then the frictional force = $\mu_K mg$
The body moves forward with an acceleration

$$a = \frac{F - \mu_k mg}{m}$$

- If a minimum force, required to move the body is applied and it is further continued, the body moves with an acceleration. $a = (\mu_s - \mu_k)g$

85. MOTION OF A BODY DOWN THE ROUGH INCLINED PLANE

- Acceleration down the plane $a = g(\sin\theta - \mu_k \cos\theta)$

  This is independent of the mass of the body
- Minimum force required to prevent the body sliding down $= mg(\sin\theta - \mu_k \cos\theta)$ (in the direction up the plane)
- If the body starts from rest from the top of the inclined plane of length l, time taken to reach the bottom of the plane t

  $$= \sqrt{\frac{2l}{g(\sin\theta - \mu_k\cos\theta)}}$$
- Velocity of the body at the bottom of the plane.

  $$v = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}$$
- Work done against the frictional force $= \mu_k mgl\cos\theta$
- Net workdone on the body $= mgl(\sin\theta - \mu_k\cos\theta)$

86. MOTION OF A BODY UP THE INCLINED PLANE

- When a body is just dragged upwards along a inclined plane then
- acceleration $a = -g(\sin\theta + \mu_k\cos\theta)$
- Time of travel $t = \sqrt{\frac{2l}{g(\sin\theta + \mu_k\cos\theta)}}$
- Work done by the driving force

  $W = (mg\sin\theta + \mu_k mg\cos\theta)L$
- Force required to drag with an acceleration 'a' is $F = \mu_k mg\cos\theta + mg\sin\theta + ma$
- Let us suppose that a car is running with a velocity 'u'. If its engine is stopped then
- Frictional force on the tyres of car is $F = \mu_k mg$
- Acceleration of the car $a = \frac{F}{m} = -\mu_k g$
- Time taken to come to rest $t = \frac{u}{\mu_k g}$
- Distance travelled before coming to rest

  $$S = \frac{u^2}{2\mu_k g}$$
- In case of a car taking a turn of radius 'r' on a horizontal level road, safe maximum speed is $V_{max} = \sqrt{\mu_s gr}$ and maximum angular velocity is $w_{max} = \sqrt{\mu_s g/r}$

  A block is placed on a truck moving along the horizontal with an acceleration 'a'. Then acceleration of the block is $a' = a - \mu g$ in the backward direction
- A chain of length L is placed on rough table such that minimum length 'l' is hanging over the edge so that the chain doesn't slip off. Then fractional length hanging over the edge is

  $$\frac{l}{L} = \frac{\mu}{\mu + 1}$$
- In a death well of radius 'r' a motor cyclist is moving on the wall in a circular path. Then the safe minimum speed v, then $v = \sqrt{\frac{gr}{\mu}}$
- A block of mass m is pressed against a wall without falling, by applying minimum horizontal force F. Then $F = \frac{mg}{\mu}$
- A block is pressed between two hands without falling, by applying minimum horizontal force 'F' by each hand. Then $F = \frac{mg}{2\mu}$
- A block is placed against the vertical side of a truck moving along the horizontal, with an acceleration 'a' without falling. Then $a = \frac{g}{\mu}$
- A block placed on the top of an incline just comes to rest at the bottom. If $(1/n)^{th}$ of the length of the incline is rough then $\mu = n\tan\theta$.
- A body released on a rough incline to reach the bottom takes time, 'n' times taken by it when it is released on the top of the same incline when it is smooth. If angle of the incline is θ then

  $$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$$